JEE MAINS MATHS TEST

Answer Key

1)	(2)	2)	(1)	3)	(1)	4)	(4)
5)	(1)	6)	(4)	7)	(3)	8)	(1)
9)	(3)	10)	(4)	11)	(4)	12)	(2)
13)	(4)	14)	(1)	15)	(1)	16)	(2)
17)	(1)	18)	(2)	19)	(3)	20)	(2)

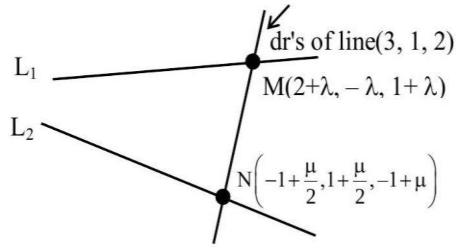
<u>Integer Answer Type</u>

21)	19	22)	9	23)	39	24)	32	25)	1

MATHEMATICS
SECTION-A

1. Ans. (2)

Sol.



$L_1 : \frac{x-2}{1} = \frac{y}{-1} = \frac{z-1}{1} = \lambda$

$L_2 : \frac{x+1}{\frac{1}{2}} = \frac{y-1}{\frac{1}{2}} = \frac{z+1}{1} = \mu$

dr of line MN will be

$< 3 + \lambda - \frac{\mu}{2}, -1 - \lambda - \frac{\mu}{2}, 2 + \lambda - \mu >$ & it will be proportional to $< 3,1,2 >$

$\therefore \frac{3 + \lambda - \frac{\mu}{2}}{3} = \frac{-1 - \lambda - \frac{\mu}{2}}{1} = \frac{2 + \lambda - \mu}{2}$

$4\lambda + \mu = -6$

$4 + 3\lambda = 0$

$\Rightarrow \lambda = -\frac{4}{3} \& \mu = -\frac{2}{3}$

$\therefore$ Coordinate of M will be $< \left(\frac{2}{3}, \frac{4}{3}, -\frac{1}{3}\right)$

and equation of required line will be.

$\frac{x - \frac{2}{3}}{3} = \frac{y - \frac{4}{3}}{1} = \frac{z + \frac{1}{3}}{2} = k$

So any point on this line will be

$\left(\frac{2}{3} + 3k, \frac{4}{3} + k, -\frac{1}{3} + 2k\right)$

$\because \frac{2}{3} + 3k = -\frac{1}{3} \Rightarrow k = -\frac{1}{3}$

$\therefore$ Point lie on the line for

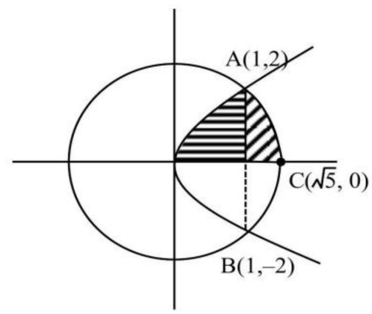
$k = -\frac{1}{3}$ is $\left(-\frac{1}{3}, 1, -1\right)$

2. Ans. (1)

Sol. $y^2 = 4x$

$x^2 + y^2 = 5$

$\therefore$ Area of shaded region as shown in the figure will be



$A_1 = \int_0^1 \sqrt{4x}\,dx + \int_1^{\sqrt{5}} \sqrt{5 - x^2}\,dx$

$= \frac{4}{3} \cdot \left[x^{\frac{3}{2}}\right]_0^1 + \left[\frac{x}{2}\sqrt{5 - x^2} + \frac{5}{2}\sin^{-1} \frac{x}{\sqrt{5}}\right]_1^{\sqrt{5}}$

$= \frac{1}{3} + \frac{5\pi}{4} - \frac{5}{2}\sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$

$\therefore$ Required Area = 2 A_1

$= \frac{2}{3} + \frac{5\pi}{2} - 5\sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$

$= \frac{2}{3} + 5\left(\frac{\pi}{2} - \sin^{-1}\frac{1}{\sqrt{5}}\right)$

$= \frac{2}{3} + 5\cos^{-1}\frac{1}{\sqrt{5}}$

$= \frac{2}{3} + 5\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$

3. Ans. (1)

Sol. $(2y - 5)\frac{dy}{dx} = -3$

$(2y - 5)dy = -3dx$

$2 \cdot \frac{y^2}{2} - 5y = -3x + \lambda$

$\because$ Curve passes through (0,1)

$\Rightarrow \lambda = -4$

$\because$ Curve will be

$\left(y - \frac{5}{2}\right)^2 = -3\left(x - \frac{3}{4}\right)$

$\therefore$ Vertex of parabola will be

$\left(\frac{3}{4}, \frac{5}{2}\right)$

$\because 2x + 3y = 9$

4. Ans. (4)

Sol.

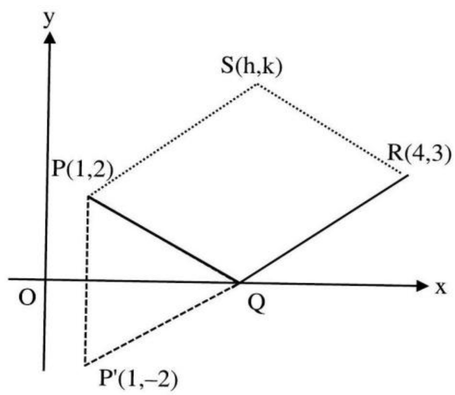


Image of P wrt x -axis will be $P'(1, -2)$
equation of line joining P'R will be

$$y - 3 = \frac{5}{3}(x - 4)$$

Above line will meet x -axis at Q where

$$y = 0 \Rightarrow x = \frac{11}{5}$$

$$\therefore Q\left(\frac{11}{5}, 0\right)$$

$\because$ PQRS is parallelogram so their diagonals will bisects each other

$$\Rightarrow \frac{4+1}{2} = \frac{\frac{11}{5} + h}{2} \& \frac{2+3}{2} = \frac{k+0}{2}$$

$$\Rightarrow h = \frac{14}{5} \& k = 5$$

$$\therefore hk^2 = \frac{14}{5} \times 5^2 = 70$$

5. Ans. (1)

Sol. $3x + 5y + \lambda z = 3$
$7x + 11y - 9z = 2$
$97x + 155y - 189z = \mu$
$93x + 155y + 31\lambda z = 93$
$97x + 155y - 189z = \mu$
$\quad - \quad - \quad + \quad -$

$-4x + (31\lambda + 189)z = 93 - \mu$

$1085x + 1705y - 1395z = 310$
$1067x + 1705y - 2079z = 11\mu$
$\quad -$

$18x + 684z = 310 - 11\mu$
$-36x + 9(31\lambda + 189)z = 9(93 - \mu)$

$36x + 1368z = 2(310 - 11\mu)$
$(279\lambda + 3069)z = 1457 - 31\mu$

for infinite solutions -

$$\lambda = \frac{-3069}{279} = \frac{-341}{31}$$

$$\mu = \frac{1457}{31}$$

$$\mu + 2\lambda = \frac{1457 - 682}{31} = \frac{775}{31} = 25$$

6. Ans. (4)

Sol. $x^2(1+x)^{98} + x^3(1+x^{97}) + x^4(1+x)^{96} + \cdots \ldots x^{54}(1+x)^{46}$

Coeff. of x^{70}: ${}^{98}C_{68} + {}^{97}C_{67} + {}^{96}C_{66} + \cdots \ldots$
${}^{47}C_{17} + {}^{46}C_{16}$
$= {}^{46}C_{30} + {}^{47}C_{30} + \cdots \ldots\ldots {}^{98}C_{30}$
$= ({}^{46}C_{31} + {}^{46}C_{30}) + {}^{47}C_{30} + \cdots \ldots\ldots {}^{98}C_{30} - {}^{46}C_{31}$
$= {}^{47}C_{31} + {}^{47}C_{30} + \cdots \ldots\ldots {}^{98}C_{30} - {}^{46}C_{31}$
......
$= {}^{99}C_{31} - {}^{46}C_{31} = {}^{99}C_{p} - {}^{46}C_{q}$

Possible values of (p + q) are 62, 83, 99, 46

$$\Rightarrow p + q = 83$$

7. Ans.(3)Sol. $\int \frac{2 - \tan x}{3 + \tan x} dx = \int \frac{2\cos x - \sin x}{3\cos x + \sin x} dx$

$2\cos x - \sin x = A(3\cos x + \sin x) + B(\cos x - 3\sin x)$

$3A + B = 2$
$A - 3B = -1$

$$\Rightarrow A = \frac{1}{2}, B = \frac{1}{2}$$

$$\therefore \int \frac{2\cos x - \sin x}{3\cos x + \sin x} dx$$

$$= \frac{x}{2} + \frac{1}{2}\ln|3\cos x + \sin x| + C$$

$$= \frac{1}{2}(x + \ln|3\cos x + \sin x|) + C$$

$$= \frac{1}{2}(\alpha x + \ln|\beta \sin x + \gamma \cos x|) + C$$

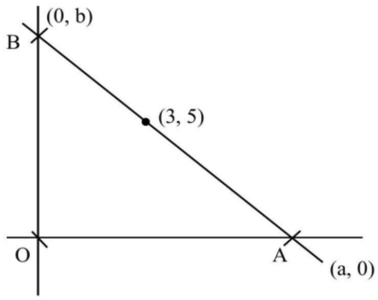
$\alpha = 1, \beta = 1, \gamma = 3$

$$\therefore \alpha + \frac{\gamma}{\beta} = 1 + \frac{3}{1} = 4$$

8. Ans. (1)

Sol. $\frac{x}{a}+\frac{y}{b}=1$

$$\frac{3}{a}+\frac{5}{b}=1 \Rightarrow b=\frac{5a}{a-3}, a>3$$



$$A=\frac{1}{2}ab=\frac{1}{2}a\frac{5a}{(a-3)}=\frac{5}{2}\cdot\frac{a^2}{a-3}$$

$$=\frac{5}{2}\left(\frac{\mathrm{a}^2-9+9}{\mathrm{a}-3}\right)$$

$$=\frac{5}{2}\left(a+3+\frac{9}{a-3}\right)$$

$$=\frac{5}{2}\left(a-3+\frac{9}{a-3}+6\right)\geq 30$$

9. Ans. (3)

Sol. We know that

$$(\cos\ \theta)\left(\cos\ (60^\circ-\theta)\right)\left(\cos\ (60^\circ+\theta)\right)=\frac{1}{4}\cos\ 3\theta$$

So equation reduces to

$$\left|\frac{1}{4}\cos\ 3\theta\right|\leq\frac{1}{8}$$

$$\Rightarrow |\cos\ 3\theta|\leq\frac{1}{2}$$

$$\Rightarrow -\frac{1}{2}\leq\cos\ 3\theta\leq\frac{1}{2}$$

$\Rightarrow$ maximum value of $\cos\ 3\theta=\frac{1}{2}$, here

$$\Rightarrow 3\theta=2\mathrm{n}\pi\pm\frac{\pi}{3}$$

$$\theta=\frac{2n\pi}{3}\pm\frac{\pi}{9}$$

As $\theta\in[0,2\pi]$ possible values are

$$\theta=\left\{\frac{\pi}{9},\frac{5\pi}{9},\frac{7\pi}{9},\frac{11\pi}{9},\frac{13\pi}{9},\frac{17\pi}{9}\right\}$$

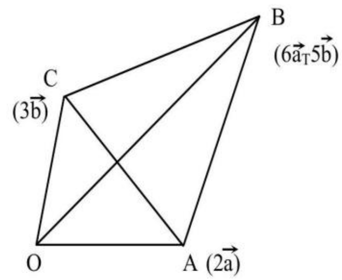
Whose sum is

$$\frac{\pi}{9}+\frac{5\pi}{9}+\frac{7\pi}{9}+\frac{11\pi}{9}+\frac{13\pi}{9}+\frac{17\pi}{9}$$

$$=\frac{54\pi}{9}=6\pi$$

10. Ans. (4)

Sol.



Area of parallelogram having sides

$\overrightarrow{OA}\&\overrightarrow{OC}=|\overrightarrow{OA}\times\overrightarrow{OC}|=|2\vec{a}\times 3\,\vec{b}|=15$

$6|\vec{a}\times\vec{b}|=15$

$$\Rightarrow |\vec{a}\times\vec{b}|=\frac{5}{2} \qquad (1)$$

Area of quadrilateral

$$\mathrm{OABC}=\frac{1}{2}\left|\vec{d}_1\times\vec{d}_2\right|$$

$$=\frac{1}{2}|\overrightarrow{AC}\times\overrightarrow{OB}|=\frac{1}{2}|(3\,\vec{b}-2\vec{a})\times(6\vec{a}+5\,\vec{b})|$$

$$=\frac{1}{2}|18\,\vec{b}\times\vec{a}-10\vec{a}\times\vec{b}|=14|\vec{a}\times\vec{b}|$$

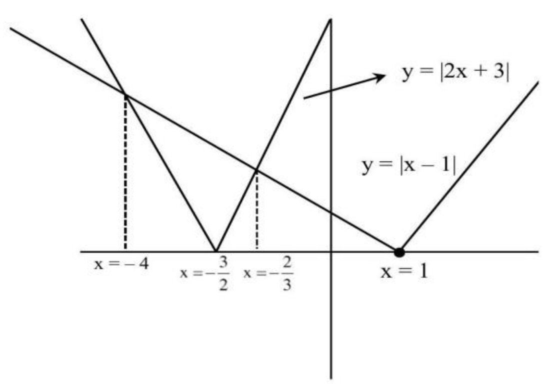
$$=14\times\frac{5}{2}=35$$

11. Ans. (4)

Sol. Domain of $f(x)=\sin^{-1}\left(\frac{x-1}{2x+3}\right)$ is

$2x+3\neq 0\&x\neq\frac{-3}{2}$ and $\left|\frac{(x-1)}{2x+3}\right|\leq 1$

$|x-1|\leq|2x+3|$



For $|2x+3| \geq |x-1|$

$x \in (-\infty, -4] \cup \left(-\frac{2}{3}, \infty\right)$

$\alpha = -4 \& \beta = -\frac{2}{3} : 12\alpha\beta = 32$

12. Ans. (2)

Sol. $\frac{1}{1\cdot(1+d)} + \frac{1}{(1+d)(1+2\,d)} + \cdots \ldots \ldots \ldots.$

$$\frac{1}{(1+9d)(1+10d)} = 5$$

$\frac{1}{d}\left[\frac{(1+d)-1}{1\cdot(1+d)} + \frac{(1+2\,d)-(1-d)}{(1+d)(1+2\,d)}\right] + .$

$$\frac{(1+10d)-(1+9d)}{(1+9d)(1+10d)} = 5$$

$\frac{1}{d}\left[\left(1-\frac{1}{1+d}\right) + \left(\frac{1}{1+d} - \frac{1}{1+2\,d}\right) + \cdots \ldots \ldots \ldots\right.$

$$\left.\left(\frac{1}{1+9d} - \frac{1}{1+10d}\right)\right] = 5$$

$$\frac{1}{d}\left[1 - \frac{1}{(1+10\,d)}\right] = 5$$

$$\frac{10\,d}{1+10\,d} = 5\,d$$

$50\,d = 5$

13. Ans. (4)

Sol. $f(x) = ax^3 + bx^2 + cx + 41$

$f'(x) = 3ax^2 + 2bx + cx$

$\Rightarrow f'(1) = 3a + 2\,b + c = 2 \quad (1)$

$f'(n) = 6ax + 2\,b$

$\Rightarrow f''(1) = 6a + 2\,b = 4$

$3a + b = 2 \quad (2)$

(1) - (2)

$b + c = 0 \quad (3)$

$f(1) = 40$

$a + b + c + 41 = 40$

use (3)

$a + 41 = 40$

by (2)

$-3 + b = 2 \Rightarrow b = 5 \& c = -5$

$a^2 + b^2 + c^2 = 1 + 25 + 25 = 51$

14. Ans. (1)

Sol. $(2+c)x + 5c^2\left(\frac{1-3x}{5}\right) = 1$

$$x = \frac{1-c^2}{2+c-3c^2}, y = \frac{1-3x}{5}$$

$$= \frac{c-1}{5(2+c-3c^2)}$$

$$h = \lim_{c\to 1}\frac{(1-c)(1+c)}{(1-c)(2+3c)} = \frac{2}{5}$$

$$K = \lim_{c\to 1}\frac{c-1}{-5(c-1)(3c+2)}$$

$$= -\frac{1}{25}$$

Centre $\left(\frac{2}{25}, -\frac{1}{25}\right)$,

$$r = \sqrt{\left(2-\frac{2}{5}\right)^2 + \left(0-\frac{1}{25}\right)^2}$$

$$= \sqrt{\frac{64}{25} + \frac{1}{625}}$$

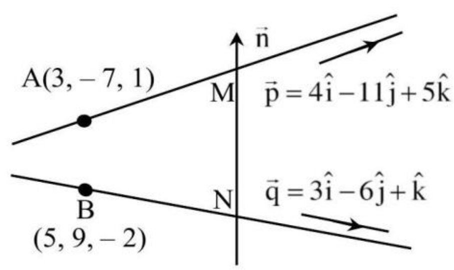
$$r = \frac{\sqrt{161}}{25}$$

$$\left(x-\frac{2}{5}\right)^2 + \left(y+\frac{1}{25}\right)^2 = \frac{161}{125}$$

$$\Rightarrow 25x^2 + 25y^2 - 20x + 2y - 60 = 0$$

15. Ans. (1)

Sol.



$\vec{n} = \vec{p} \times \vec{q}$

$$\vec{n} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 4 & -11 & 5 \\ 3 & -6 & 1 \end{vmatrix} = 19\hat{\imath} + 11\hat{\jmath} + 9\hat{k}$$

S.d. = projection of $\overrightarrow{AB}$ on $\vec{n}$

$$= \left|\frac{\overrightarrow{AB}\cdot\vec{n}}{|\vec{n}|}\right|$$

$$= \left|\frac{(2\hat{\imath} + 16\hat{\jmath} - 3\hat{k})\cdot(19\hat{\imath} + 11\hat{\jmath} + 9\hat{k})}{\sqrt{361+121+81}}\right|$$

$$= \frac{38+176-27}{\sqrt{563}}$$

S.d. $= \frac{187}{\sqrt{563}}$

16. Ans. (2)

Sol. $x + y = 10$

Median $= 18 = M$

M.D. $= \frac{\sum f_i|x_i - M|}{\sum f_i}$

$$1.25=\frac{36+x+2y}{40}$$

$$x+2y=14 \qquad (1)$$

by (1) & (2)

$x=6, y=4$

$\Rightarrow 4x+5y=24+20=44$

$\text{Age}(x_i)$	f	$\lvert x_i-M\rvert$	$f_i\lvert x_i-M\rvert$
15	5	3	15
16	8	2	16
17	5	1	5
18	12	0	0
19	x	1	x
20	y	2	2 y

17. Ans. (1)

Sol. $(x^2+y^2)dx=5xydy$

$\Rightarrow \frac{dy}{dx}=\frac{x^2+y^2}{5xy}$

Put $y=Vx$

$\Rightarrow V+x\frac{dv}{dx}=\frac{1+V^2}{5V}$

$\Rightarrow \frac{xdv}{dx}=\frac{1-4V^2}{5V}$

$\Rightarrow \int \frac{V}{1-4V^2}dV=\int \frac{dx}{5x}$

Let $1-4V^2=t$

$\Rightarrow -8\,VdV=dt$

$\Rightarrow \int \frac{dt}{(-8)(t)}=\int \frac{dx}{5x}$

$\Rightarrow \frac{-1}{8}\ln|t|=\frac{1}{5}\ln|x|+\ln C$

$\Rightarrow -5\ln|t|=8\ln|x|+\ln K$

$\Rightarrow \ln X^8+\ln|t^5|+\ln K=0$

$\Rightarrow x^8|t^5|=C$

$\Rightarrow x^8|1-4V^2|^5=C$

$\Rightarrow x^8\left|\frac{x^2-4y^2}{x^2}\right|^5=C$

$\Rightarrow |x^2-4y^2|^5=Cx^2$

given $y(1)=0$

$\Rightarrow |1|^5=C\Rightarrow C=1$

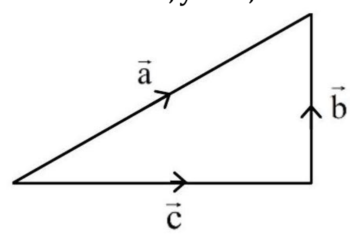
$\Rightarrow |x^2-4y^2|^5=x^2$

18. Ans. (2)

Sol. $\vec{c}=\vec{a}-\vec{b}$

$\Rightarrow (x,y,z)=(\alpha-5,1,-2)$

$\Rightarrow x=\alpha-5, y=1, z=-2 \qquad (1)$



Area of $\Delta=5\sqrt{6}$ (given)

$\frac{1}{2}|\vec{\mathbf{a}}\times\vec{\mathbf{c}}|=5\sqrt{6}$

$$\left\|\begin{matrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ \alpha & 4 & 2\\ x & 1 & -2\end{matrix}\right\|=10\sqrt{6}$$

$\Rightarrow |-10\hat{\imath}-\hat{\jmath}(-2\alpha-2x)+\hat{k}(\alpha-4x)|$
$=10\sqrt{6}$

$\Rightarrow (2\alpha+2\alpha-10)^2+(\alpha-4\alpha+20)^2$
$=500$

$\Rightarrow (4\alpha-10)^2+(20-3\alpha)^2=500$

$\Rightarrow 25\alpha^2-80\alpha-120\alpha=0$

$\Rightarrow \alpha(25\alpha-200)=0$

$\Rightarrow \alpha=8$ (given α is + ve number)

$\Rightarrow x=\alpha-5=3$

$|\vec{c}|^2=x^2+y^2+z^2$

$=9+1+4$

$=14$

19. Ans. (3)

Sol. $x^2+2\sqrt{2}x-1=0$

$\alpha+\beta=-2\sqrt{2}$

$\alpha\beta=-1$

$\alpha^4+\beta^4=(\alpha^2+\beta^2)^2-2\alpha^2\beta^2$

$=((\alpha+\beta)^2-2\alpha\beta)^2-2(\alpha\beta)^2$

$=(8+2)^2-2(-1)^2$

$=100-2=98$

$\alpha^6+\beta^6=(\alpha^3+\beta^3)^2-2\alpha^3\beta^3$

$=((\alpha+\beta)((\alpha+\beta)^2-3\alpha\beta)^2$
$-2(\alpha\beta)^3$

$=(-2\sqrt{2}(8+3))^2+2$

$=(8)(121)+2=970$

$\frac{1}{10}(\alpha^6+\beta^6)=97$

$x^2-(98+97)x+(98)(97)=0$

$\Rightarrow x^2-195x+9506=0$

20. Ans. (2)

Sol. $f(x) = x^2 + 9$ $g(x) = \frac{x}{x-9}$

$a = f(g(10)) = f\left(\frac{10}{10-9}\right)$

$= f(10) = 109$

$b = g(f(3)) = g(9 + 9)$

$= g(18) = \frac{18}{9} = 2$

$E: \frac{x^2}{109} + \frac{y^2}{2} = 1$

$e^2 = 1 - \frac{2}{109} = \frac{107}{109}$

$\ell = \frac{2(2)}{\sqrt{109}} = \frac{4}{\sqrt{109}}$

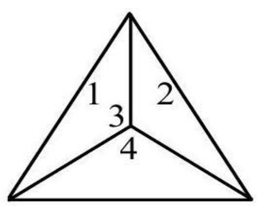
$8e^2 + \ell^2 = \frac{8(107)}{109} + \frac{16}{109}$

$= 8$

SECTION-B

21. Ans. (19)

Sol. a, b, c ∈ {1,2,3,4}



Tetrahedral dice

$ax^2 + bx + c = 0$

has all real roots

$\Rightarrow D \geq 0$

$\Rightarrow b^2 - 4ac \geq 0$

Let $b = 1 \Rightarrow 1 - 4ac \geq 0$ (Not feasible)

$b = 2 \Rightarrow 4 - 4ac \geq 0$

$1 \geq ac \Rightarrow a = 1, c = 1,$

$b = 3 \Rightarrow 9 - 4ac \geq 0$

$\frac{9}{4} \geq ac$

$\Rightarrow a = 1, c = 1$

$\Rightarrow a = 1, c = 2$

$\Rightarrow a = 2, c = 1$

$b = 4 \Rightarrow 16 - 4ac \geq 0$

$4 \geq ac$

$\Rightarrow a = 1, c = 1$

$\Rightarrow a = 1, c = 2$ $\Rightarrow a = 2, c = 1$

$\Rightarrow a = 1, c = 3$ $\Rightarrow a = 3, c = 1$

$\Rightarrow a = 1, c = 4$ $\Rightarrow a = 4, c = 1$

$\Rightarrow a = 2, c = 2$

Probability $= \frac{12}{(4)(4)(4)} = \frac{3}{16} = \frac{m}{m}$

$m + n = 19$

22. Ans. (9)

Sol. $|z - 1| \leq 1$

$\Rightarrow |(x-1) + iy| \leq 1$

$\Rightarrow \sqrt{(x-1)^2 + y^2} \leq 1$

$\Rightarrow (x-1)^2 + y^2 \leq 1$ (1)

Also $|z - 5| \leq |z - 5i|$

$(x-5)^2 + y^2 \leq x^2 + (y-5)^2$

$-10x \leq -10y$

$\Rightarrow x \geq y$ (2)

Solving (1) and (2)

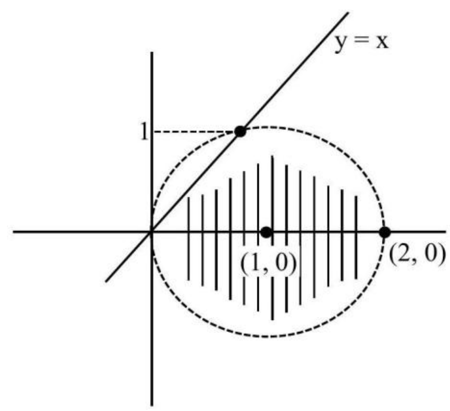
$\Rightarrow (x-1)^2 + x^2 = 1$

$\Rightarrow 2x^2 - 2x = 0$

$\Rightarrow x(x-1) = 0$

$\Rightarrow x = 0$ or $x = 1$

$y = 0$ or $y = 1$



Given $x, y \in I$

Points (0,0), (1,0), (2,0), (1,1), (1, −1)

to find

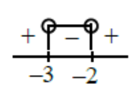
$|z_1|^2 + |z_2|^2 + |z_3|^2 + |z_4|^2 + |z_5|^2$

$= 0 + 1 + 4 + 1 + 1 + 1 + 1 = 9$

23. Ans. (39)

Sol. $\frac{x^2+x+2}{x^2+5x+6} < 0$

$\Rightarrow \frac{1}{(x+2)(x+3)} < 0$



$x \in (-3, -2)$

$f(x) = 1 + x(\lambda^2 - x^2)$ (1)

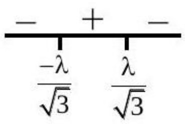
Finding local minima

$f'(x) = (\lambda^2 - x^2) + (-2x)\cdot x$

Put $f'(x) = 0$

$\Rightarrow \lambda^2 = 3x^2$

$\Rightarrow x = \pm\dfrac{\lambda}{\sqrt{3}}$



Local min Local max

We want local min

$\Rightarrow x = \dfrac{-\lambda}{\sqrt{3}}$

from (1)

$x \in (-3, -2)$

$-3 < \dfrac{-\lambda}{\sqrt{3}} < -2$

$3\sqrt{3} > \lambda > 2\sqrt{3}$

$\alpha = 2\sqrt{3}, \beta = 3\sqrt{3}$

$\alpha^2 + \beta^2 = 12 + 27 = 39$

24. Ans. (32)

Sol. $\sum_{r=1}^{\infty} \frac{n}{\sqrt{n^4+r^4}} - \frac{2nr^2}{(n^2+r^2)\sqrt{n^4+r^4}}$

$$\sum_{r=1}^{\infty} \frac{\frac{1}{n}}{\sqrt{1+\left(\frac{r}{n}\right)^4}} - \frac{2\left(\frac{1}{n}\right)\left(\frac{r}{n}\right)^2}{\left(1+\left(\frac{r}{n}\right)^2\right)\sqrt{1+\left(\frac{r}{n}\right)^4}}$$

$$\Rightarrow \int_0^1 \frac{dx}{\sqrt{1+x^4}} - \frac{2x^2dx}{(1+x^2)\sqrt{1+x^4}}$$

$$\Rightarrow \int_0^1 \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}}dx$$

$$\Rightarrow \int_0^1 \frac{\frac{1}{x^2}-1}{\left(x+\frac{1}{x}\right)\sqrt{x^2+\frac{1}{x^2}}}dx$$

$$\Rightarrow -\int_0^1 \frac{1-\frac{1}{x^2}}{\left(x+\frac{1}{x}\right)\sqrt{\left(x+\frac{1}{x}\right)^2-2}}dx$$

$x + \frac{1}{x} = t \Rightarrow 1 - \frac{1}{x^2}dx = dt$

$$\Rightarrow -\int_\infty^2 \frac{dt}{t\sqrt{t^2-2}}$$

$$\Rightarrow -\int_\infty^2 \frac{tdt}{t^2\sqrt{t^2-2}}$$

take $t^2 - 2 = \alpha^2$

$tdt = \alpha d\alpha$

$$\Rightarrow -\int_\infty^{\sqrt{2}} \frac{\alpha\, d\alpha}{(\alpha^2+2)\alpha}$$

$$\Rightarrow -\int_\infty^{\sqrt{2}} \frac{d\alpha}{\alpha^2+2}$$

$$\Rightarrow \frac{-1}{\sqrt{2}}\tan^{-1}\frac{\alpha}{\sqrt{2}}\Big]_\infty^{\sqrt{2}}$$

$$\Rightarrow \frac{-1}{\sqrt{2}}\{\tan^{-1} 1\} + \frac{1}{\sqrt{2}}\tan^{-1}\infty$$

$$\Rightarrow \frac{1}{\sqrt{2}}\left\{\frac{\pi}{2} - \frac{\pi}{4}\right\}$$

$$\Rightarrow \frac{\pi}{4\sqrt{2}} = \frac{\pi}{K}$$

So $K = 4\sqrt{2}$

$K^2 = 32$

25. Ans. (1)

Sol. $(428)^{2024} = (420+8)^{2024}$

$= (21\times 20 + 8)^{2024}$

$= 21\text{ m} + 8^{2024}$

Now $8^{2024} = (8^2)^{1012}$

$= (64)^{1012}$

$= (63+1)^{1012}$

$= (21\times 3 + 1)^{1012}$

$= 21n + 1$

$\Rightarrow$ Remainder is