JEE MAINS MATHS TEST

(9 APRIL 2024 – 1ST SHIFT)

MATHEMATICS

SECTION-A

1.Let the line L intersect the lines
$x-2=-y=z-1, 2(x+1)=2(y-1)=z+1$
and be parallel to the line $\frac{x-2}{3}=\frac{y-1}{1}=\frac{z-2}{2}$. Then which of the following points lies on L ?
(1) $\left(-\frac{1}{3}, 1, 1\right)$
(2) $\left(-\frac{1}{3}, 1, -1\right)$
(3) $\left(-\frac{1}{3}, -1, -1\right)$
(4) $\left(-\frac{1}{3}, -1, 1\right)$

2. The parabola $y^2=4x$ divides the area of the circle $x^2+y^2=5$ in two parts. The area of the smaller part is equal to :
(1) $\frac{2}{3}+5\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$
(2) $\frac{1}{3}+5\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$
(3) $\frac{1}{3}+\sqrt{5}\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$
(4) $\frac{2}{3}+\sqrt{5}\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$

3. The solution curve, of the differential equation $2y\frac{dy}{dx}+3=5\frac{dy}{dx}$, passing through the point
(0,1) is a conic, whose vertex lies on the line :
(1) $2x+3y=9$
(2) $2x+3y=-9$
(3) $2x+3y=-6$
(4) $2x+3y=6$

4. A ray of light coming from the point $P(1,2)$ gets reflected from the point Q on the x - axis and then passes through the point

$R(4,3)$. If the point $S(h, k)$ is such that PQRS is a parallelogram, then hk^2 is equal to :
(1) 80
(2) 90
(3) 60
(4) 70

5. Let $\lambda, \mu \in R$. If the system of equations
$3x+5y+\lambda z=3$

$7x + 11y - 9z = 2$
$97x + 155y - 189z = \mu$
has infinitely many solutions, then $\mu + 2\lambda$ is equal to :
(1) 25
(2) 24
(3) 27
(4) 22

6. The coefficient of x^{70} in $x^2(1+x)^{98} + x^3(1+x)^{97} + x^4(1+x)^{96} + \cdots \ldots\ldots + x^{54}(1+x)^{46}$ is ${}^{99}C_p - {}^{46}C_q$.
Then a possible value to p + q is :
(1) 55
(2) 61
(3) 68
(4) 83

7. Let
$\int \frac{2-\tan x}{3+\tan x} dx = \frac{1}{2}(\alpha x + \log_e |\beta \sin x + \gamma \cos x|) + C$, where C is the constant of integration.

Then $\alpha + \frac{\gamma}{\beta}$ is equal to :
(1) 3
(2) 1
(3) 4
(4) 7

8. A variable line L passes through the point (3,5) and intersects the positive coordinate axes at the points A and B. The minimum area of the triangle OAB, where O is the origin, is:
(1) 30
(2) 25
(3) 40
(4) 35

9. Let
$|\cos\theta \cos(60-\theta)\cos(60-\theta)| \leq \frac{1}{8}, \theta \in [0,2\pi]$
Then, the sum of all $\theta \in [0,2\pi]$, where $\cos 3\theta$ attains its maximum value, is :
(1) 9π
(2) 18π
(3) 6π
(4) 15π

10. Let $\overrightarrow{OA} = 2\vec{a}, \overrightarrow{OB} = 6\vec{a} + 5\vec{b}$ and $\overrightarrow{OC} = 3\vec{b}$, where O is the origin. If the area of the parallelogram with adjacent sides $\overrightarrow{OA}$ and $\overrightarrow{OC}$ is 15 sq. units, then the area (in sq. units) of the quadrilateral OABC is equal to :
(1) 38

(2) 40
(3) 32
(4) 35

11. If the domain of the function
$f(x)=\sin^{-1}\left(\frac{x-1}{2x+3}\right)$ is $R-(\alpha,\beta)$
then $12\alpha\beta$ is equal to:
(1) 36
(2) 24
(3) 40
(4) 32

12. If the sum of series
$$\frac{1}{1\cdot(1+d)}+\frac{1}{(1+d)(1+2d)}+\cdots\ldots\ldots+\frac{1}{(1+9d)(1+10d)}$$
is equal to 5 , then 50 d is equal to:
(1) 20
(2) 5
(3) 15
(4) 10

13. Let $f(x)=ax^3+bx^2+ex+41$ be such that $f(1)=40, f'(1)=2$ and $f''(1)=4$.
Then $a^2+b^2+c^2$ is equal to :
(1) 62
(2) 73
(3) 54
(4) 51

14. Let a circle passing through (2,0) have its centre at the point (h, k). Let (X_c, y_c) be the point of intersection of the lines $3x+5y=1$ and $(2+c)x+5c^2y=1$. If $h=\lim_{c\to1}x_c$ and $k=\lim_{c\to1}y_c$, then the equation of the circle is :
(1) $25x^2+25y^2-20x+2y-60=0$
(2) $5x^2+5y^2-4x-2y-12=0$
(3) $25x^2+25y^2-2x+2y-60=0$
(4) $5x^2+5y^2-4x+2y-12=0$

15. The shortest distance between the line
$\frac{x-3}{4}=\frac{y+7}{-11}=\frac{z-1}{5}$ and $\frac{x-5}{3}=\frac{y-9}{-6}=\frac{z+2}{1}$ is :
(1) $\frac{187}{\sqrt{563}}$
(2) $\frac{178}{\sqrt{563}}$
(3) $\frac{185}{\sqrt{563}}$
(4) $\frac{179}{\sqrt{563}}$

16. The frequency distribution of the age of students in a class of 40 students is given below.

Age	15	16	17	18	19	20
No. of Students	5	8	5	12	x	y

If the mean deviation about the median is 1.25 , then $4x + 5y$ is equal to :
(1) 43
(2) 44
(3) 47
(4) 46

17. The solution of the differential equation $(x^2 + y^2)dx - 5xydy = 0, y(1) = 0$, is :
(1) $|x^2 - 4y^2|^5 = x^2$
(2) $|x^2 - 2y^2|^6 = x$
(3) $|x^2 - 4y^2|^6 = x$
(4) $|x^2 - 2y^2|^5 = x^2$

18. Let three vectors $\vec{a} = \alpha\hat{\imath} + 4\hat{\jmath} + 2\hat{k}$, $\vec{b} = 5\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$, $\vec{c} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ from a triangle such that $\vec{c} = \vec{a} - \vec{b}$ and the area of the triangle is $5\sqrt{6}$. if α is a positive real number, then $|\vec{c}|^2$ is :
(1) 16
(2) 14
(3) 12
(4) 10

19. Let α, β be the roots of the equation $x^2 + 2\sqrt{2}x - 1 = 0$. The quadratic equation, whose roots are $\alpha^4 + \beta^4$ and $\frac{1}{10}(\alpha^6 + \beta^6)$, is :
(1) $x^2 - 190x + 9466 = 0$
(2) $x^2 - 195x + 9466 = 0$
(3) $x^2 - 195x + 9506 = 0$
(4) $x^2 - 180x + 9506 = 0$

20. Let $f(x) = x^2 + 9$, $g(x) = \frac{x}{x-9}$ and $a = fog(10)$, $b = gof(3)$. If e and 1 denote the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{a} + \frac{y^2}{b} = 1$, then $8e^2 + l^2$ is equal to.
(1) 16
(2) 8
(3) 6
(4) 12

SECTION-B

21.Let a, b and c denote the outcome of three independent rolls of a fair tetrahedral die, whose
four faces are marked 1,2,3,4. If the probability that $\mathrm{ax}^2+\mathrm{bx}+\mathrm{c}=0$ has all real roots is $\frac{\mathrm{m}}{\mathrm{n}}$, $\gcd(\mathrm{m},\mathrm{n})=1$, then $\mathrm{m}+\mathrm{n}$ is equal to .

22. The sum of the square of the modulus of the elements in the set $\{\mathrm{z}=\mathrm{a}+\mathrm{ib}: \mathrm{a},\mathrm{b}\in\mathrm{Z}, \mathrm{z}\in\mathrm{C}, |\mathrm{z}-1|\le 1, |\mathrm{z}-5|\le|\mathrm{z}-5\mathrm{i}|\}$ is .

23. Let the set of all positive values of λ, for which the point of local minimum of the function
$\left(1+x(\lambda^2-x^2)\right)$ satisfies $\frac{x^2+x+2}{x^2+5x+6}<0$, be (α,β).
Then $\alpha^2+\beta^2$ is equal to .

24. Let
$\lim_{n\to\infty}\left(\frac{n}{\sqrt{n^4+1}}-\frac{2n}{(n^2+1)\sqrt{n^4+1}}+\frac{n}{\sqrt{n^4+16}}-\frac{8n}{(n^2+4)\sqrt{n^4+16}}+\cdots\ldots.+\frac{n}{\sqrt{n^4+n^4}}-\frac{2n\cdot n^2}{(n^2+n^2)\sqrt{n^4+n^4}}\right)$ be $\frac{\pi}{k}$, using only the principal values of the inverse trigonometric functions. Then k^2 is equal to .

25. The remainder when 428^{2024} is divided by 21 is