JEE MAINS MATHS TEST

ANSWER KEY

1)	1	2)	2	3)	4	4)	1
5)	3	6)	2	7)	1	8)	4
9)	4	10)	3	11)	3	12)	1
13)	4	14)	1	15)	3	16)	4
17)	3	18)	1	19)	2	20)	4

Integer Answer Type

21)	15	22)	96	23)	56	24)	63
25)	30						

MATHEMATICS

SECTION-A

1.Ans. (1)

Sol. $\vec{p} = 2\hat{i} + 3\hat{j} + 4\hat{k}, \vec{q} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

$$\Rightarrow \vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = -\hat{i} + 2\hat{j} - \hat{k}$$

$A \equiv (1,2,3) B \equiv (\lambda, 4,5)$

Shortest Distance $= \left|\frac{\overrightarrow{AB}\cdot(\vec{P}\times\vec{q})}{|\vec{P}\times\vec{q}|}\right|$

$$\frac{1}{\sqrt{6}} = \left|\frac{((\lambda-1)\hat{\imath} + 2\hat{\jmath} + 2\hat{k}) \cdot (-\hat{\imath} + 2\hat{\jmath} - \hat{k})}{\sqrt{6}}\right|$$

$\Rightarrow |-\lambda + 1 + 4 - 2| = 1 \Rightarrow |\lambda - 3| = 1$

$\Rightarrow \lambda = 3 \pm 1 = 4,2$

Radius of circle passing through points

$(0,0), (4,2) \& (2,4)$

$$= \frac{abc}{4\Delta} = \frac{\sqrt{20} \times \sqrt{20} \times \sqrt{8}}{4 \times \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ 0 & 4 & 2 \\ 0 & 2 & 4 \end{vmatrix}} = \frac{20 \times 2\sqrt{2}}{2 \times 12}$$

$$= \frac{5\sqrt{2}}{3}$$

2.Ans. (2)

Sol. $x^2 + x + 1 = 0$

α is root

$\therefore \alpha^2 + \alpha + 1 = 0$

$\Rightarrow \alpha = \omega$ as ω^2 [cube root of unity]

also

$[4 - a - 2\,b \quad 64 - a - 14\,b \quad 52 + 2a - 8\,b]$

$= [0 \quad 0 \quad 0]$

$\therefore a + 2b = 4$
$a + 14b = 64$
$\Rightarrow 12b = 60 \Rightarrow b = 5$
$\Rightarrow a = -6$
$\therefore \frac{4}{\alpha^4} + \frac{m}{\alpha^{-6}} + \frac{n}{\alpha^5} = 3$
$\Rightarrow \frac{4}{\omega} + \frac{m}{1} + \frac{n}{\omega^2} = 3$
$\Rightarrow 4\omega^2 + m + n\omega = 3$
$\Rightarrow 4\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) + m + n\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = 3$

$$\therefore -2 + \mathrm{m} - \frac{\mathrm{n}}{2} = 3 \qquad (1)$$

$\& \frac{-4\sqrt{3}}{2} + \frac{n\sqrt{3}}{2} = 0$
$\therefore \mathrm{n} = 4$

$$\mathrm{m} = 7$$

$\therefore \mathrm{m} + \mathrm{n} = 11$

3.Ans. (4)

Sol. $\mathrm{f(x)} = \frac{\mathrm{x}}{3} + \frac{3}{\mathrm{x}} + 3, \mathrm{x} \neq 0$
$f'(x) = \frac{1}{3} - \frac{3}{x^2} = 0 \ \Rightarrow x = \pm 3$
$f'(x) = \frac{x^2 - 3}{3x^2}$
$\mathrm{f'(x)} > 0 \forall (-\infty, -3) \cup (3, \infty) \rightarrow$ increasing
$\mathrm{f'(x)} < 0 \forall (-3,0) \cup (0,3) \rightarrow$ decreasing
$\sum_{i=1}^{5} \alpha_i^2 = (-3)^2 + (3)^2 + (-3)^2 + (0)^2 + (3)^2$

$$= 36$$

4.Ans. (1)

Sol. $\mathrm{P(A)} = \frac{7}{10}, \mathrm{P(B)} = \frac{4}{10}$
$\mathrm{P(A \cup \overline{B})} = \frac{5}{10}$
$P\left(\frac{B}{A \cup \bar{B}}\right) = \frac{P(B \cap (A \cup \bar{B}))}{P(A \cup \bar{B})}$
$= \frac{\mathrm{P((B \cap \overline{B}) \cup (B \cap A))}}{\mathrm{P(A \cup \overline{B})}} = \frac{\mathrm{P(A \cap B)}}{\mathrm{P(A \cup \overline{B})}}$

$$=\frac{P(A)-P(A\cap\overline{B})}{P(A)+P(\overline{B})-P(A\cap\overline{B})}=\frac{\frac{7}{10}-\frac{5}{10}}{\frac{7}{10}+\left(1-\frac{4}{10}\right)-\frac{5}{10}}$$

$$=\frac{2}{8}=\frac{1}{4}$$

5.Ans. (3)

Sol. If $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\cdots..\infty=\frac{\pi^4}{90}$

$\beta=\frac{1}{2^4}+\frac{1}{4^4}+\frac{1}{6^4}+\cdots,$

$=\frac{1}{16}\left[\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\cdots..\right],$

$=\frac{1}{16}\times\frac{\pi^4}{90}$ using (ii)

$$\alpha=\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+\cdots..\infty \qquad (ii)$$

$$\left(\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\frac{1}{4^4}+\frac{1}{5^4}+\cdots..\right)$$

$$-\left(\frac{1}{2^4}+\frac{1}{4^4}+\frac{1}{6^4}+\cdots..\right)$$

$\alpha=\frac{\pi^4}{90}-\frac{1}{16}\times\frac{\pi^4}{90}$ [using (i) and (ii)]

$$\alpha=\frac{16-1}{16\times 90}\times\pi^4=\frac{15}{16\times 90}\pi^4=\frac{\pi^4}{96}$$

$$\therefore\frac{\alpha}{\beta}=\frac{\frac{\pi^4}{96}}{\frac{\pi^4}{16\times 90}}=\frac{16\times 90}{96}=15$$

6.Ans. (2)

Sol. $|x-2|^2+2|x-2|-|x-2|-2=0$

$\Rightarrow (|x-2|+2)(|x-2|-1)=0$

$\Rightarrow |x-2|=1$

$\Rightarrow x=2\pm 1=3,1$

$\Rightarrow$ sum of square of roots $=9+1=10$

$x^2-2|x-3|-5=0$

Case-I $x-3\geq 0$

$\Rightarrow x^2-2x+1=0$

$\Rightarrow (x-1)^2=0$

$\Rightarrow x=1$

But $x\geq 3$

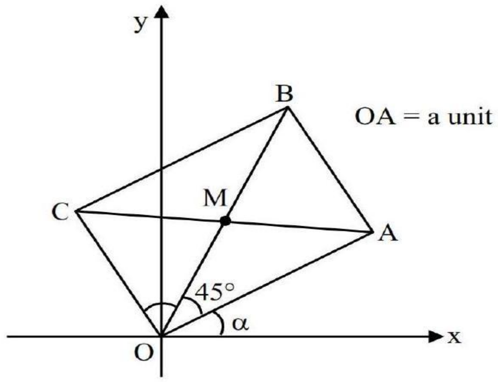
$\Rightarrow x\in\phi$

Case-II $x-3<0$

$x^2 + 2x - 11 = 0, D > 0 \Rightarrow$ Real & distinct roots
$f(x) = x^2 + 2x - 11$
$f(3) > 0, \dfrac{-p}{2a} = -1 < 3$
$\Rightarrow$ both roots < 3, both roots acceptable
Sum of square of roots $= (\alpha + \beta)^2 - 2\alpha\beta$
$= 4 + 22 = 26$
$\Rightarrow$ Final sum $= 10 + 26 = 36$

7.Ans. (1)

Sol.



Slope of diagonal OB $= \frac{\sqrt{3}+1}{1-\sqrt{3}}$

$$= \tan\ 105°$$

$\therefore \alpha = 60°$
$\therefore \mathrm{A}(\mathrm{a}\cos 60°, \mathrm{a}\sin 60°)$
$\therefore \mathrm{A}\left(\dfrac{\mathrm{a}}{2}, \dfrac{\sqrt{3}\mathrm{a}}{2}\right)$
A Lies on other diagonal
$\therefore \left(\dfrac{\sqrt{3}-1}{2}\right)a - \left(\dfrac{\sqrt{3}+1}{2}\right) \cdot \sqrt{3}a + 8\sqrt{3} = 0$
$\mathrm{a}\left[\dfrac{\sqrt{3}-1-3-\sqrt{3}}{2}\right] = -8\sqrt{3}$
$a = 4\sqrt{3}$
$\therefore \mathrm{a}^2 = 48$

8.Ans. (4)

Sol. $I_1 = \int_{-\frac{1}{2}}^{1} 2xf(2x(1-2x))dx$

$\Rightarrow 2x = t \Rightarrow 2dx = dt \ \Rightarrow I_1 = \frac{1}{2}\int_{-1}^{2} tf(t(1-t))dt$

$\Rightarrow 2I_1 = \int_{-1}^{2}(1-t)f(1-t)(1-(1-t))dt$

$\Rightarrow 2I_1 = \int_{-1}^{2} f\Big(t(1-t)dt - \int_{-1}^{2} tf(t(1-t)dt$

$\Rightarrow 2I_1 = I_2 - 2I_1$

$\Rightarrow 4I_1 = I_2$

$\Rightarrow \frac{I_2}{I_1} = 4$

9.Ans. (4)

Sol. Let vector $\vec{p}$ in plane of $\vec{a}\&\vec{b} = K(\vec{a} + \lambda\vec{b})$

$\vec{p} \perp \vec{a} = \vec{p} \cdot \vec{a} = 0$

$\Rightarrow K(\vec{a} + \lambda\vec{b}) \cdot \vec{a} = 0$

$\Rightarrow \vec{a} \cdot \vec{a} + \lambda\vec{b} \cdot \vec{a} = 0$

$\Rightarrow 6 + \lambda(3) = 0$

$\Rightarrow \lambda = -2$

$\Rightarrow \vec{p} = (-3\hat{\imath} + 3\hat{k})$

Unit vector $\rightarrow \pm\frac{(-\hat{\imath}+\hat{k})}{\sqrt{2}}$

10.Ans. (3)

Sol. $E: \frac{x^2}{4/3} + \frac{y^2}{4/P} = 1$

Centre of circle (1,2), radius

$r = \sqrt{1+4+11}$

$r = 4$

$\because$ E pass from centre (1,2)

$\therefore \frac{3}{4} + P = 1$

$P = \frac{1}{4}$ $\therefore$ vertical ellipse

$e = \sqrt{1 - \frac{4/3}{16}} = \sqrt{1 - \frac{1}{12}} = \sqrt{\frac{11}{12}}$

$\therefore$ Focal distance of C (h, k)

$= b \pm ek$

$F_1 = 4 + \sqrt{\frac{11}{12}} \times 2$

$F_2 = 4 - \sqrt{\frac{11}{12}} \times 2$

$\therefore F_1 F_2 = 16 - \frac{11}{3} = \frac{37}{3}$

$\therefore 6 F_1 F_2 - r = 74 - 4 = 70$

11.Ans. (3)

Sol. Let, $I = \pi^2 \int_{-1}^{3/2} |x\sin \pi x| dx$

$= \pi^2 \left\{ \int_{-1}^{1} x\sin \pi x dx - \int_{1}^{3/2} x\sin \pi x dx \right\}$

$= \pi^2 \left\{ 2\int_{0}^{1} x\sin \pi x dx - \int_{-1}^{3/2} x\sin \pi x dx \right\}$

Consider

$\int x\sin \pi x dx$

$-x \cdot \frac{1}{\pi} \cos \pi x + \int 1 \cdot \frac{1}{\pi} \cos \pi x dx$

$= -\frac{x}{\pi} \cos \pi x + \frac{\sin \pi x}{\pi^2}$

$I = \pi^2 \left\{ 2\left(-\frac{x}{\pi} \cos \pi x + \frac{\sin \pi x}{\pi^2} \right)_0^1 - \left(-\frac{x}{\pi} \cos \pi x + \frac{\sin \pi x}{\pi^2} \right)_1^{3/2} \right\}$

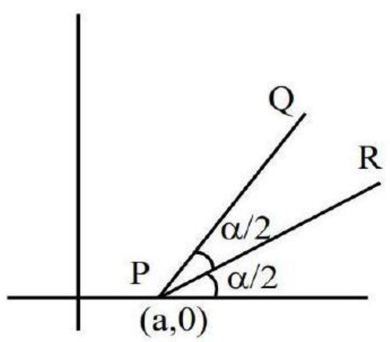
$= \pi^2 \left\{ \frac{2}{\pi} - \left(-\frac{1}{\pi^2} - \frac{1}{\pi} \right) \right\}$

$= \pi^2 \left\{ \frac{3}{\pi} + \frac{1}{\pi^2} \right\}$

$= 3\pi + 1$

12.Ans. (1)

Sol.



$m_{PR} = 2 - \sqrt{3} = \tan 15°$

$\therefore \dfrac{\alpha}{2} = 15^\circ \Rightarrow \alpha = 30^\circ$

equation of PR :

$y = \tan\ 15^\circ(x - a)$

$y = (2 - \sqrt{3})(x - a)$

$\perp$ distance from origin $= \frac{1}{\sqrt{2}}$

$$\left|\frac{\sqrt{3}a - 2a}{\sqrt{4 + 3 - 4\sqrt{3} + 1}}\right| = \frac{1}{\sqrt{2}}$$

$$\frac{|a|(2 - \sqrt{3})}{2\sqrt{(2 - \sqrt{3})}} = \frac{1}{\sqrt{2}}$$

$$|a| = \frac{\sqrt{2}}{\sqrt{2 - \sqrt{3}}} = \sqrt{2}(\sqrt{2 + \sqrt{3}})$$

$a^2 = 2(2 + \sqrt{3})$

$3a^2 \tan^2\ \alpha - 2\sqrt{3}$

$$3 \times (4 + 2\sqrt{3}) \cdot \frac{1}{3} - 2\sqrt{3} = 4$$

13.Ans. (4)

Sol. ${}^{12}C_3 - {}^{5}C_3 = 210$

14.Ans. (1)

Sol. $1 + 10\mathrm{Re}\left(\frac{2\cos\ \theta + i\sin\ \theta}{\cos\ \theta - 3i\sin\ \theta}\right) = 0$

$\therefore z + \bar{z} = 2\mathrm{Re}(z)$

$$\frac{2\cos\ \theta + \mathrm{i}\sin\ \theta}{\cos\ \theta - 3\mathrm{i}\sin\ \theta} + \frac{2\cos\ \theta - \mathrm{i}\sin\ \theta}{\cos\ \theta + 3\mathrm{i}\sin\ \theta} = 2 \times \left(\frac{-1}{10}\right)$$

$$\frac{(2\cos^2\ \theta - 3\sin^2\ \theta) + (2\cos^2\ \theta) - (3\sin^2\ \theta)}{\cos^2\ \theta + 9\sin^2\ \theta} = \frac{-2}{10}$$

$$\Rightarrow \frac{2\cos^2\ \theta - 3\sin^2\ \theta}{\cos^2\ \theta + 9\sin^2\ \theta} = \frac{-1}{10}$$

$\Rightarrow 20\cos^2\ \theta - 30\sin^2\ \theta = -\cos^2\ \theta - 9\sin^2\ \theta$

$21\cos^2\ \theta - 21\sin^2\ \theta = 0$

$\Rightarrow \cos\ 2\theta = 0$

$$2\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$$

$$\Rightarrow \sum\theta^2 = \frac{\pi^2}{16} + \frac{9\pi^2}{16} + \frac{25\pi^2}{16} + \frac{49\pi^2}{16} = \frac{84\pi^2}{16} = \frac{21\pi^2}{4}$$

15.Ans. (3)

Sol. $A = \{0,1,2,3,4,5\}$

$R \equiv \{(0,3),(3,0),(0,4),(4,0),(1,3),(3,1),(1,4),$
$(4,1),(2,3),(3,2),(2,4),(4,2),(3,3),(3,4),(4,3),(4,4)\}$
Total 16 elements
Not reflexive as $(0,0),\ldots\ldots,(2,2) \notin \mathrm{R}$
Symmetric $\because$ $\forall$ all a,b
$(a,b)\&(b,a) \in R$
Not transitive $\because$ $(0,3),(3,1) \in R$
but $(0,1) \notin \mathrm{R}$
$\Rightarrow$ Only S_2 correct

16.Ans. (4)
Sol. $T_r = {}^{1016}C_r(5)^{\frac{1016-r}{2}}7^{\frac{r}{8}}$
$\Rightarrow \mathrm{r} = 0,8,16,24,\ldots,1016$
$1016 = 0 + (\mathrm{n}-1)8$
$$\Rightarrow \mathrm{n}-1 = \frac{1016}{8} = 127$$
So, $n = 128$.

17.Ans. (3)
Sol. $f(x) = x - 1$
$\mathrm{f(f(x)) = f(x) - 1 = x - 1 - 1 = x - 2}$
$g(f(f(x))) = e^{x-2}$
$$\therefore \frac{dy}{dx} = \mathrm{e}^{-2\sqrt{x}} \times \mathrm{e}^{\mathrm{x}-2} - \frac{1}{\sqrt{\mathrm{x}}}\mathrm{y}$$
$\frac{dy}{dx} + \frac{1}{\sqrt{x}}y = e^{x-2\sqrt{x}-2}$ which is L.D.E
I.F. $= e^{\int \frac{dy}{\sqrt{x}}} = e^{2\sqrt{x}}$

Its solution is
$y \times e^{2\sqrt{x}} = \int e^{2\sqrt{x}} \times e^{x-2\sqrt{x}-2}dx + c$
$y \times e^{2\sqrt{x}} = \int e^{x-2}dx + c$
$y \times e^{2\sqrt{x}} = e^{x-2} + c$
Given $\mathrm{x} = 0, \mathrm{y} = 0 \Rightarrow 0 = \mathrm{e}^{-2} + \mathrm{c}\,; \mathrm{c} = -\mathrm{e}^{-2}$
$\therefore \mathrm{y} \times \mathrm{e}^{2\sqrt{\mathrm{x}}} = \mathrm{e}^{\mathrm{x}-2} - \mathrm{e}^{-2}$
when $\mathrm{x} = 1, \mathrm{y} \times \mathrm{e}^2 = \mathrm{e}^{-1} - \mathrm{e}^{-2}$
$$y = \frac{e^{-1} - e^{-2}}{e^2} = \frac{\frac{1}{e} - \frac{1}{e^2}}{e^2} = \frac{e^2 - e}{e^5} = \frac{e-1}{e^4}$$
Option (1) is correct

18.Ans. (1)

Sol. $\cot^{-1}\left(\frac{|\sec\ 2|-1}{\tan\ 2}\right)-\cot^{-1}\left(\frac{\left|\sec\frac{1}{2}\right|+1}{\tan\frac{1}{2}}\right)$

$=\cot^{-1}\left(\frac{-1-\cos\ 2}{\sin\ 2}\right)-\cot^{-1}\left(\frac{1+\cos\ \frac{1}{2}}{\sin\ \frac{1}{2}}\right)$

$=\pi-\cot^{-1}(\cot\ 1)-\cot^{-1}\left(\cot\ \frac{1}{4}\right)$

$=\pi-1-\frac{1}{4}=\pi-\frac{5}{4}$

19.Ans. (2)

Sol. $|A|=\begin{vmatrix}2 & 2+p & 2+p+q\\ 4 & 6+2p & 8+3p+2q\\ 6 & 12+3p & 20+6p+3q\end{vmatrix}$

$\mathrm{C}_3\rightarrow\mathrm{C}_3-\mathrm{C}_2-\mathrm{C}_1\times\frac{\mathrm{q}}{2}$

Then $\mathrm{C}_3\rightarrow\mathrm{C}_2-\mathrm{C}_1\mathrm{X}\left(1+\frac{\mathrm{p}}{2}\right)$

$\Rightarrow|\mathrm{A}|=\begin{vmatrix}2 & 0 & 0\\ 4 & 2 & 2+\mathrm{p}\\ 6 & 6 & 8+3\mathrm{p}\end{vmatrix}$

$\Rightarrow|A|=2(16+6p-12-6p)=8=2^3$

$|\mathrm{adj}(\mathrm{adj}(3\ \mathrm{A}))|=|3\ \mathrm{A}|^{(3-1)^2}=|3\ \mathrm{A}|^4$

$=(3^3|\ \mathrm{A}|)^4=(3^3\times2^3)^4=2^{12}\times3^{12}$

$\Rightarrow\mathrm{m}+\mathrm{n}=24$

20.Ans. (4)

Sol. $\lim_{x\rightarrow0}\frac{\tan^{-1}\ x+\frac{1}{2}[\ln\ (1+x)-\ln\ (1-x)]-2x}{x^5}$

$=\lim_{x\rightarrow0}\frac{\left(x-\frac{x^3}{3}+\frac{x^5}{5}\ldots.\right)+\frac{1}{2}\left[x-\frac{x^2}{2}+\frac{x^3}{3}\ldots-\left(-x-\frac{x^2}{2}-\frac{x^3}{3}\ldots\right)\right]-2x}{x^5}$

$=\lim_{x\rightarrow0}\frac{2x+\frac{2x^5}{5}\ldots-2x}{x^5}=\frac{2}{5}$

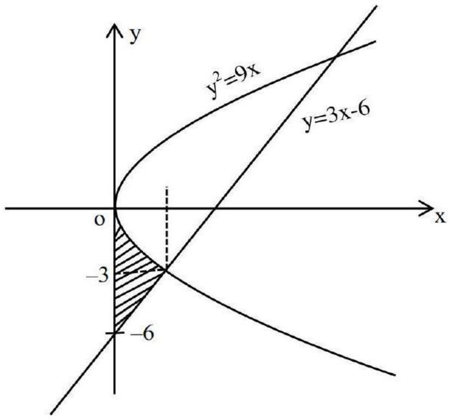
$\lim_{x\rightarrow1}x^{\frac{2}{(1-x)}}=\mathrm{e}^{\lim_{x\rightarrow1}\left(\frac{2}{1-x}\right)(x-1)}=\mathrm{e}^{-2}$

$\Rightarrow$ Both statements correct

SECTION-B

21.Ans. (15)

Sol. $0 \leq 9x \leq y^2 \& y \geq 3x - 6$



$A =$ Required Area $= \left[\int_0^1 (-3\sqrt{x})dx - \int_0^1 (3x-6)dx\right]$

$A = -3\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right)\Bigg|_0^1 - \left(\frac{3x^2}{2} - 6x\right)\Bigg|_0^1$

$A = -2[1-0]\left[\frac{3}{2} - 6\right]$

$\mathrm{A} = -2 - \frac{3}{2} + 6 = \frac{5}{2}$ Sq. unit

$\therefore 6\,\mathrm{A} = 6 \times \frac{5}{2} = 15$

22.Ans. (96)

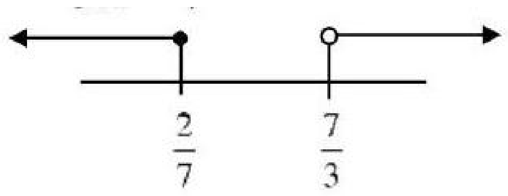
Sol. $f(x) = \cos^{-1}\left(\frac{4x+5}{3x-7}\right)$

$\Rightarrow -1 \leq \left(\frac{4x+5}{3x-7}\right) \leq 1$

$\left(\frac{4x+5}{3x-7}\right) \geq -1$

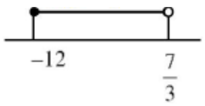
$\frac{4\mathrm{x}+5+3\mathrm{x}-7}{3\mathrm{x}-7} \geq 0$

$\Rightarrow \frac{7\mathrm{x}-2}{3\mathrm{x}-7} \geq 0$



$x \in \left(-\infty, \frac{2}{7}\right] \cup \left(\frac{7}{3}, \infty\right)$

$\& \frac{4x+5}{3x-7} \leq 1 \Rightarrow \frac{x+12}{3x-7} \leq 0$



$\therefore$ Domain of f(x) is

$\left[-12, \frac{2}{7}\right] \alpha = -12, \beta = \frac{2}{7}$

$g(x) = \log_2(2 - 6\log_{27}(2x+5))$

Domain

$2 - 6\log_{27}(2x+5) > 0$

$\Rightarrow\ 6\log_{27}(2x+5) < 2$

$\Rightarrow\ \log_{27}(2x+5) < \frac{1}{3}$

$\Rightarrow\ 2x + 5 < 3$

$\Rightarrow\ x < -1$

$\& 2x + 5 > 0 \Rightarrow x > -\frac{5}{2}$

Domain is $x \in \left(-\frac{5}{2}, -1\right)$

$\gamma = -\frac{5}{2}, \delta = -1$

$|7(\alpha+\beta) + 4(\gamma+\delta)| = \left|7\left(-12 + \frac{2}{7} + 4\left(-\frac{5}{2} - 1\right)\right|\right.$

$|-82 - 14| = 96$

23.Ans. (56)

Sol. $L_1: x + 2 = y - 1 = z = \ell$

$L_2: \frac{x-3}{5} = \frac{y}{-1} = \frac{z-1}{1} = m$

$L_3: \frac{x}{-3} = \frac{y-3}{5} = \frac{z-2}{1} = n$

Point of intersection of L_1 and L_2

$\ell - 2 = 5m + 3$

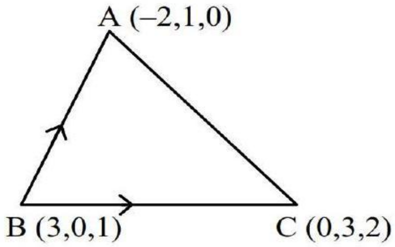
$\left.\begin{array}{r}\ell + 1 = -m \\ \ell = m + 1\end{array}\right\} \ell = 0, m = -1\ A(-2,1,0)$

Point of intersection of L_2 and L_3

$$\left.\begin{array}{l}5m+3=-3n\\-m=3n+3\\m+1=n+2\end{array}\right\} m=0, n=-1, B(3,0,1)$$

Point of intersection L_3 and L_4

$$\left.\begin{array}{r}-3\mathrm{n}=\ell-2\\3\mathrm{n}+3=\ell+1\\\mathrm{n}+2=\ell\end{array}\right\} \ell=2, \mathrm{n}=0, \mathrm{C}(0,3,2)$$



$$\mathrm{Ar}(\triangle \mathrm{ABC})=\left|\frac{1}{2}\right|\begin{array}{ccc}\hat{\mathrm{i}} & \hat{\mathrm{j}} & \hat{\mathrm{k}}\\-5 & 1 & -1\\-3 & 3 & 1\end{array}\Bigg|\Bigg|$$

$$\mathrm{A}=\frac{1}{2}|\hat{\mathrm{i}}(4)-\hat{\mathrm{j}}(-8)+\hat{\mathrm{k}}(-12)|$$

$$A=\frac{1}{2}\sqrt{16+64+144}=\sqrt{56}$$

$$\mathrm{A}^2=56$$

24.Ans. (63)

Sol. $(1919)^{1919}=(1920-1)^{1919}$

$= {}^{1919}\mathrm{C}_0(1920)^{1919}- {}^{1919}\mathrm{C}_1(1920)^{1918}+\cdots..$

$+{}^{1919}\mathrm{C}_{1918}(1920)^{\,1}- {}^{1919}\mathrm{C}_{1919}$

$=100\lambda+1919\times 1920-1$

$=100\lambda+3684480-1$

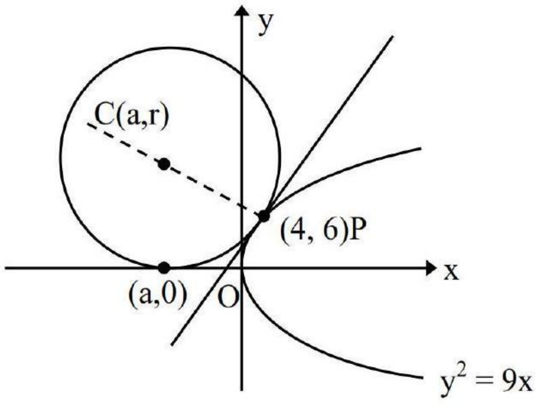
$=100\lambda+\cdots\ldots\ldots..79$ (last two digit)

⇒ Number having last two digit 79

∴ Product of last two digit 63

25.Ans. (30)

Sol.



$(x-a)^2+(y-r)^2=r^2$
$(4-a)^2+(6-r)^2=r^2$
$16+a^2-8a+36+r^2-12r=r^2$
$a^2-8a-12r+52=0$
Tangent to parabola at (4,6) is
$6.4=9.\left(\frac{x+4}{2}\right)$ i.e. $3x-4y+12=0$
This is also tangent to the circle
$\therefore \mathrm{CP}=\mathrm{r}$
$\frac{3a-4r+12}{5}=\pm r$

$$3a+12=4\mathrm{r}\pm 5\mathrm{r}\{\begin{smallmatrix}\mathrm{ar}\\ -\mathrm{r}\end{smallmatrix} \quad (1)$$

equation of circle is

$$(x-a)^2+(y-r)^2=r^2 \quad (2)$$

satsty $P(4,6) \Rightarrow a^2-8a-12r+52=0$
From equation (1)
If $\mathrm{a}+4=3\mathrm{r}$ then $\mathrm{a}=+6$ (rejected)
If $3a+12=-r$ then $a=-14$ and $r=30$