MATHEMATICS
SECTION-A

1.Let the values of λ for which the shortest distance between the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-\lambda}{3}=\frac{y-4}{4}=\frac{z-5}{5}$ is $\frac{1}{\sqrt{6}}$ be λ_1 and λ_2. Then the radius of the circle passing through the points $(0,0),(\lambda_1,\lambda_2)$ and (λ_2,λ_1) is

(1) $\frac{5\sqrt{2}}{3}$
(2) 4
(3) $\frac{\sqrt{2}}{3}$
(4) 3

2.Let α be a solution of $x^2+x+1=0$, and for some a and b in $\mathbb{R}$, $[4 \quad \mathrm{a} \quad \mathrm{b}]\begin{bmatrix} 1 & 16 & 13 \\ -1 & -1 & 2 \\ -2 & -14 & -8 \end{bmatrix}=[0 \quad 0 \quad 0]$. If $\frac{4}{\alpha^4}+\frac{\mathrm{m}}{\alpha^{\mathrm{a}}}+\frac{\mathrm{n}}{\alpha^{\mathrm{b}}}=3$, then $\mathrm{m}+\mathrm{n}$ is equal to

(1) 3
(2) 11
(3) 7
(4) 8

3. Let the function $f(x)=\frac{x}{3}+\frac{3}{x}+3, x\neq 0$ be strictly increasing in $(-\infty,\alpha_1)\mathrm{U}(\alpha_2,\infty)$ and strictly decreasing in $(\alpha_3,\alpha_4)\cup(\alpha_4,\alpha_5)$. Then $\sum_{i=1}^{5}\alpha_i^2$ is equal to :-

(1) 48
(2) 28
(3) 40
(4) 36

4.If A and B are two events such that $P(A)=0.7$, $\mathrm{P(B)}=0.4$ and $\mathrm{P(A}\cap\overline{\mathrm{B}})=0.5$, where $\overline{\mathrm{B}}$ denotes the complement of B, then $P(B \mid (A\cup\bar{B}))$ is equal:-

(1) $\frac{1}{4}$
(2) $\frac{1}{2}$
(3) $\frac{1}{6}$
(4) $\frac{1}{3}$

5. If $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\cdots..\infty=\frac{\pi^4}{90}$,
$\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+\cdots..\infty=\alpha$,
$\frac{1}{2^4}+\frac{1}{4^4}+\frac{1}{6^4}+\cdots..\infty=\beta$,
then $\frac{\alpha}{\beta}$ is equal to
(1) 23
(2) 18
(3) 15
(4) 14

6. The sum of the squares of the roots of $|\mathrm{x}+2|^2+|\mathrm{x}-2|-2=0$ and the squares of the roots of $x^2-2|x-3|-5=0$, is
(1) 26
(2) 36
(3) 30
(4) 24

7. Let a be the length of a side of a square OABC with O being the origin. Its side OA makes an acute angle α with the positive x -axis and the equations of its diagonals are $(\sqrt{3}+1)x+(\sqrt{3}-1)y=0$ and $(\sqrt{3}-1)x-(\sqrt{3}+1)y+8\sqrt{3}=0$. Then a^2 is equal to
(1) 48
(2) 32
(3) 16
(4) 24

8. Let $f(x)$ be a positive function and
$I_1=\int_{-\frac{1}{2}}^{1}2xf(2x(1-2x))dx$ and $I_2=\int_{-1}^{2}f(x(1-x))dx$.
Then the value of $\frac{I_2}{I_1}$ is equal to -
(1) 9
(2) 6
(3) 12
(4) 4

9. Let $\vec{\mathrm{a}}=\hat{\imath}+2\hat{\jmath}+\hat{\mathrm{k}}$ and $\vec{\mathrm{b}}=2\hat{\imath}+\hat{\jmath}-\hat{\mathrm{k}}$. Let $\hat{\mathrm{c}}$ be a unit vector in the plane of the vectors $\vec{a}$ and $\vec{b}$ and be perpendicular to $\vec{a}$. Then such a vector $\hat{c}$ is :
(1) $\frac{1}{\sqrt{5}}(\hat{\jmath}-2\hat{k})$
(2) $\frac{1}{\sqrt{3}}(-\hat{\imath}+\hat{\jmath}-\hat{k})$
(3) $\frac{1}{\sqrt{3}}(\hat{\imath}-\hat{\jmath}+\hat{k})$
(4) $\frac{1}{\sqrt{2}}(-\hat{\imath}+\hat{\mathrm{k}})$

10. Let the ellipse $3x^2 + py^2 = 4$ pass through the centre C of the circle $x^2 + y^2 - 2x - 4y - 11 = 0$ of radius r. Let f_1, f_2 be the focal distances of the point C on the ellipse. Then $6f_1f_2 - r$ is equal to
(1) 74
(2) 68
(3) 70
(4) 78

11. The integral $\int_{-1}^{\frac{3}{2}} (|\pi^2 x\sin(\pi x)|)dx$ is equal to :
(1) $3 + 2\pi$
(2) $4 + \pi$
(3) $1 + 3\pi$
(4) $2 + 3\pi$

12. A line passing through the point $P(a, \theta)$ makes an acute angle α with the positive x -axis. Let this line be rotated about the point P through an angle $\frac{\alpha}{2}$ in the clock-wise direction. If in the new position, the slope of the line is $2 - \sqrt{3}$ and its distance from the origin is $\frac{1}{\sqrt{2}}$, then the value of $3a^2\tan^2 \alpha - 2\sqrt{3}$ is
(1) 4
(2) 6
(3) 5
(4) 8

13. There are 12 points in a plane, no three of which are in the same straight line, except 5 points which are collinear. Then the total number of triangles that can be formed with the vertices at any three of these 12 points is
(1) 230
(2) 220
(3) 200
(4) 210

14. Let A =
$\left\{\theta \in [0,2\pi]: 1 + 10\mathrm{Re}\left(\frac{2\cos\theta + i\sin\theta}{\cos\theta - 3i\sin\theta}\right) = 0\right\}$.
Then $\sum_{\theta \in A} \theta^2$ is equal to
(1) $\frac{21}{4}\pi^2$
(2) $8\pi^2$
(3) $\frac{27}{4}\pi^2$
(4) $6\pi^2$

15. Let $A = \{0,1,2,3,4,5\}$. Let R be a relation on A defined by $(x, y) \in R$ if and only if $\max\{x, y\} \in \{3,4\}$. Then among the statements

(S_1) : The number of elements in R is 18 , and
(S_2) : The relation R is symmetric but neither reflexive nor transitive
(1) both are true
(2) both are false
(3) only (S_2) is true
(4) only (S_1) is true

16. The number of integral terms in the expansion of $\left(5^{\frac{1}{2}}+7^{\frac{1}{8}}\right)^{1016}$ is
(1) 127
(2) 130
(3) 129
(4) 128

17. Let $f(x)=x-1$ and $g(x)=e^{x}$ for $x \in \mathbb{R}$. If $\frac{dy}{dx}=\left(e^{-2\sqrt{x}} g(f(f(x)))-\frac{y}{\sqrt{x}}\right), y(0)=0$, then $y(1)$ is :-
(1) $\frac{1-e^{2}}{e^{4}}$
(2) $\frac{2e-1}{e^{3}}$
(3) $\frac{e-1}{e^{4}}$
(4) $\frac{1-e^{3}}{e^{4}}$

18. The value of $\cot^{-1}\left(\frac{\sqrt{1+\tan^{2}(2)}-1}{\tan(2)}\right)-\cot^{-1}\left(\frac{\sqrt{1+\tan^{2}\left(\frac{1}{2}\right)}+1}{\tan\left(\frac{1}{2}\right)}\right)$ is equal to
(1) $\pi-\frac{5}{4}$
(2) $\pi-\frac{3}{2}$
(3) $\pi+\frac{3}{2}$
(4) $\pi+\frac{5}{2}$

19. Let $A=\begin{bmatrix} 2 & 2+p & 2+p+q \\ 4 & 6+2p & 8+3p+2q \\ 6 & 12+3p & 20+6p+3q \end{bmatrix}$.

If $\det(\operatorname{adj}(\operatorname{adj}(3A)))=2^{m} \cdot 3^{n}, m, n \in \mathbb{N}$, then $m+n$ is equal to
(1) 22
(2) 24
(3) 26
(4) 20

20. Given below are two statements :

Statement I:

$$\lim_{x\to 0}\left(\frac{\tan^{-1} x+\log_e\sqrt{\frac{1+x}{1-x}}-2x}{x^5}\right)=\frac{2}{5}$$

Statement II : $\lim_{x\to I}\left(x^{\frac{2}{1-x}}\right)=\frac{1}{e^2}$

In the light of the above statements, choose the correct answer from the options given below :

(1) Statement I is false but Statement II is true

(2) Statement I is true but Statement II is false

(3) Both Statement I and Statement II are false

(4) Both Statement I and Statement II are true

SECTION-B

21. Let the area of the bounded region $\{(x,y): 0 \le 9x \le y^2, y \ge 3x-6\}$ be A. Then 6A is equal to

22. Let the domain of the function $f(x)=\cos^{-1}\left(\frac{4x+5}{3x-7}\right)$ be $[\alpha,\beta]$ and the domain of $g(x)=\log_2(2-6\log_{27}(2x+5))$ be (γ,δ).

Then $|7(\alpha+\beta)+4(\gamma+\delta)|$ is equal to

23. Let the area of the triangle formed by the lines $x+2=y-1=z, \frac{x-3}{5}=\frac{y}{-1}=\frac{z-1}{1}$ and $\frac{x}{-3}=\frac{y-3}{3}=\frac{z-2}{1}$ be A. Then A^2 is equal to Type equation here.

24. The product of the last two digits of $(1919)^{1919}$ is

25. Let r be the radius of the circle, which touches x -axis at point $(a,0), a<0$ and the parabola $y^2=9x$ at the point (4,6). Then r is equal to